MATH 201: LINEAR ALGEBRA
HOMEWORK DUE TUESDAY WEEK 6

Problem 1. Carry out the matrix multiplications

$$\begin{pmatrix} i & 1-i \\ 1+i & 2i \end{pmatrix}\begin{pmatrix} 1 & 3 \\ i & 2 \end{pmatrix}, \quad \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix}(3,1,1), \quad \text{and} \quad \begin{pmatrix} 3 & -1 \\ 1 & 6 \end{pmatrix}\begin{pmatrix} 5 \\ 2 \end{pmatrix},$$

where the fields of scalars are $\mathbb{C}, \mathbb{Q}$, and $\mathbb{Z}/7\mathbb{Z}$, respectively.

Problem 2. The *transpose* of a matrix A is the matrix A^t defined by $(A^t)_{ij} = A_{ji}$.
(a) Show that the rows of A^t are the columns of A, and vice versa.
(b) Show that if A and B are matrices for which the product AB is defined, then B^tA^t is defined and $(AB)^t = B^tA^t$.

Problem 3. Let A be an $m \times n$ matrix and B an $n \times p$ matrix. Let $b_1, \ldots, b_p$ be the columns of B, labeled from left to right and regarded as $n \times 1$ matrices. Show that the columns of AB are $Ab_1, \ldots, Ab_p$.