**MATH 201 HOMEWORK ASSIGNMENT 8**

**Problem 1.** Let $T : \mathbb{Q}^3 \to \mathbb{Q}^3$ be the linear transformation defined by

$$T(x, y, z) = (x - y + 2z, 2x + y, -x - 2y + 2z).$$

Determine the rank and the nullity of $T$.

**Problem 2.**

(a) Let $V$ be an $n$-dimensional vector space, where $n$ is odd. Show that there is no linear transformation $V \to V$ whose kernel and image are identical.

(b) Give an example of a linear transformation $\mathbb{R}^2 \to \mathbb{R}^2$ whose kernel and image are identical.

**Problem 3.** Describe explicitly a linear transformation from $\mathbb{R}^3$ to $\mathbb{R}^3$ whose image is the subspace spanned by $(1, 0, -1)$ and $(1, 2, 2)$.