MATH 201: LINEAR ALGEBRA
HOMEWORK DUE TUESDAY WEEK 4

Problem 1. Let V be a finite-dimensional vector space, and let U be a subspace of V.

(a) Prove that U is finite-dimensional.

(b) Prove that $\dim(U) \leq \dim(V)$.

(c) Prove that if $\dim(U) = \dim(V)$, then $U = V$.

(d) Prove that the only subspaces of $\mathbb{R}^2$ are $\{0\}, \mathbb{R}^2$, and all lines through the origin.

Problem 2. Prove that $\mathbb{R}$ is not finite-dimensional as a vector space over $\mathbb{Q}$. (Hint: the set of real numbers is *uncountable*, so there is no bijection $\mathbb{R} \leftrightarrow \mathbb{Q}$, and more generally, no bijection $\mathbb{R} \leftrightarrow \mathbb{Q}^n$ for any positive integer n.)