MATH 201: LINEAR ALGEBRA
HOMEWORK DUE FRIDAY WEEK 4

Problem 1. Consider the matrix $M = \begin{pmatrix} 1 & 3 & 2 \\ 3 & 2 & 2 \\ 5 & 1 & 2 \end{pmatrix}$.

(a) Determine a basis for the row space of M. Is the vector $(7\ 7\ 6)$ in the row space of M?

(b) Determine a basis for the column space of M. Is the vector $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ in the column space of M?

Problem 2. Answer the following questions about rank, proving your assertions.

(a) Which matrices have rank 0?
(b) Which matrices have rank 1?
(c) What is the maximal rank of an $m \times n$ matrix?

Problem 3. For each of the following functions, determine (with proof) whether or not the function is a linear transformation.

(a) $f : \mathbb{R}^2 \to \mathbb{R}$ given by $(x, y) \mapsto x + 2y$.
(b) $g : \mathbb{R} \to \mathbb{R}$ given by $x \mapsto 3x + 1$.
(c) $d/dx : D \to \mathbb{R}^{\mathbb{R}}$ given by $f \mapsto df/dx = f'$ where $D \subseteq \mathbb{R}^{\mathbb{R}}$ is the subspace of everywhere differentiable functions $\mathbb{R} \to \mathbb{R}$. (You may assume that D is a subspace.)
(d) $\mathrm{ev}_0 : \mathcal{L}(V, W) \to W$ given by $f \mapsto f(0)$ where V, W are F-vector spaces and $\mathcal{L}(V, W)$ is the F-vector space of linear transformations $V \to W$. (We call ev_0 the *evaluation at* 0 function.)