MATH 201: LINEAR ALGEBRA
HOMEWORK DUE FRIDAY WEEK 3

Problem 1. In each of the cases, write v as a linear combination of the linearly independent vectors in B. (You do not need to prove that B is linearly independent.) The ambient vector space should be clear from context.

(a) $v = (-8, 5)$, $B = \{(1, 0), (0, 1)\}$
(b) $v = (-8, 5)$, $B = \{(5, 2), (2, 1)\}$
(c) $v = x^2 + 3x + 2$, $B = \{1, x - 1, (x - 1)^2\}$
(d) $v : \{1, 2, 3\} \to \mathbb{R}$ with $v(1) = -2$, $v(2) = 1$, $v(3) = \sqrt{2}$, $B = \{\chi_1, \chi_2, \chi_3\}$ with $\chi_i(j) = 1$ if $i = j$ and $\chi_i(j) = 0$ if $i \neq j$.

Problem 2. Determine whether each of the following statements is true or false. If true, prove it; if false, provide a counterexample.

(a) Every subset of a linearly dependent set is linearly dependent.
(b) A vector space cannot have more than one basis.
(c) The span of the empty set is the empty set.
(d) If S is a linearly dependent set, then every vector in S is a linear combination of the other vectors in S.

Problem 3. For each of the following vector spaces, find a basis and prove that it is a basis.

(a) The F-vector space $U = \{a + bx + cx^2 + dx^3 \mid a + 4b - 3c + d = 0\}$ considered as a subspace of $F[x]$, the vector space of polynomials with coefficients in $\mathbb{Q}$.
(b) The F-vector space $V = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \mid b = c \right\}$ considered as a subspace of $M_{2\times 2}(F)$.
(c) The F-vector space $\mathrm{Sym}_n(F) = \{A \in M_{n\times n}(F) \mid A_{i,j} = A_{j,i} \text{ for } 1 \leq i, j \leq n\}$ of "symmetric" matrices inside $M_{n\times n}(F)$. (This generalizes (b).)