MATH 201: LINEAR ALGEBRA
HOMEWORK DUE TUESDAY WEEK 2

Problem 1. Let F be a field and let V be a vector space over F. Use the axioms listed in the definition of a vector space to prove the following properties.

(1) (Uniqueness of the zero vector) There is only one vector $0 \in V$ with the property that $w + 0 = w$ for every $w \in V$.

(2) (Uniqueness of additive inverse) For every $v \in V$ there is a unique vector w such that $v + w = 0$. This vector w will be denoted by $-v$.

(3) (a) $0v = 0$ for every $v \in V$. (Here the 0 on the left denotes the zero scalar in F while the 0 on the right denotes the 0 vector in V).
   (b) $a0 = 0$ for every $a \in F$. (Here 0 denotes the zero vector in V).
   (c) $(-1)v = -v$ for every $v \in V$.

Problem 2. Let V be the set of all strictly positive real numbers. Define addition and scalar multiplication on V as follows:

$$x \oplus y = xy \text{ for all } x, y \in V, \text{ and } a \otimes x = x^a \text{ for all } a \in \mathbb{R}, x \in V.$$

Show that V is a vector space over $\mathbb{R}$.

Problem 3. Let $V = \mathbb{R}^2$ be the set of all ordered pairs of real numbers. Define addition and scalar multiplication on V as follows:

$$(u, v) \oplus (x, y) = (u + x, 0), \text{ and } a \otimes (x, y) = (ax, ay).$$

Under these operations, V is not a vector space over $\mathbb{R}$. Which of the vector space axioms fail to hold?