MATH 201: LINEAR ALGEBRA
HOMEWORK DUE FRIDAY WEEK 2

Problem 1. In each of the following instances, v is a vector in a vector space V and S is a subset of V. Determine whether or not v is in the span of S by either expressing v as a linear combination of elements of S or showing that no such linear combination exists.

(a) $V = \mathbb{R}^2$, $S = \{(5,2),(2,1)\}$, $v = (49,18)$

(b) $V = \mathbb{Q}[x]$ (polynomials in x with coefficients in $\mathbb{Q}$), $S = \{x^2+5x-1, -2x^4+x^3+3x^2-4, -3x^3+2x^2\}$, $v = x^4+x^3+x^2+x+1$

(c) $V = M_{2\times 2}(\mathbb{C})$ (2×2 matrices with entries in $\mathbb{C}$), $S = \left\{\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}, \begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}\right\}$, $v = \begin{pmatrix}1 & 2\\ 2 & 0\end{pmatrix}$

Problem 2. Let $\mathbb{R}^{\mathbb{R}}$ denote the $\mathbb{R}$-vector space consisting of functions $f : \mathbb{R} \to \mathbb{R}$. (The vector space structure is given by $(f+g)(x) = f(x) + g(x)$ and $(cf)(x) = c(f(x))$.) Consider the following subsets of $\mathbb{R}^{\mathbb{R}}$ and determine (with proof) whether they are subspaces.

(a) The set of even functions $\mathscr{E} = \{f \in \mathbb{R}^{\mathbb{R}} \mid f(x) = f(-x) \text{ for all } x \in \mathbb{R}\}$.

(b) The set of odd functions $\mathscr{O} = \{f \in \mathbb{R}^{\mathbb{R}} \mid -f(x) = f(-x) \text{ for all } x \in \mathbb{R}\}$.

(c) The set of functions which vanish at 0, $V = \{f \in \mathbb{R}^{\mathbb{R}} \mid f(0) = 0\}$.

(d) The set of functions which have limit 0 as $x \to 0$, $L = \{f \in \mathbb{R}^{\mathbb{R}} \mid \lim_{x\to 0} f(x) = 0\}$.

(e) The set of functions which take the value 1 at 0, $W = \{f \in \mathbb{R}^{\mathbb{R}} \mid f(0) = 1\}$.