MATH 201: LINEAR ALGEBRA
HOMEWORK DUE FRIDAY WEEK 1

Problem 1. For each of the following systems of linear equations,

(i) find the associated augmented matrix M,
(ii) compute the reduced row echelon form E for M (show your work as in class, specifying your row operations), and
(iii) from E, determine whether there are solutions to the system; if there is a unique solution, state it; if there are infinitely many solutions, express the solutions in two ways: (1) parametrically, and (2) in vector form.

(a)

$$\begin{aligned} -2x + y + z &= 1 \\ 2x - y - 4z &= 0 \\ 3x - y + 3z &= 1 \end{aligned}$$

(b)

$$\begin{aligned} 2x + 2y + 6z &= 6 \\ -2x + 2y + 2z &= -2 \\ 4x + 6y + 16z &= 8 \end{aligned}$$

(c)

$$\begin{aligned} 2x + y - 2z &= 5 \\ y - z &= -1 \\ x - y + z &= 5 \end{aligned}$$

(d)

$$\begin{aligned} x - y - \frac{3}{2} &= -1 \\ \frac{3}{2} - \frac{3}{2}y - z + \frac{5}{2}w &= \frac{7}{2} \\ \frac{1}{2}x - \frac{1}{2}y - z - \frac{1}{2}w &= -\frac{3}{2} \end{aligned}$$

(e)

$$\begin{aligned} x + y + 3z &= 4 \\ x + 2y + 4z &= 5 \end{aligned}$$

Problem 2. Some questions about conics.

(a) Let $y = px^2 + qx + r$ be the equation of a general parabola. By solving a system of equations, find constants p, q, and r so that the resulting parabola passes through the points $(2, -15)$, $(-3, -1)$, and $(-11, -2)$. Is this the only such parabola?
(b) A *real plane conic* is a set of points of the form

$$\{(x, y) \in \mathbb{R}^2 \mid ax^2 + bxy + cy^2 + dx + ey + f = 0\}$$

where a, b, c, d, e, f are constants in $\mathbb{R}$, not all 0.
   (i) What happens to a real plane conic if a, b, c, d, e, f are replaced by $\lambda a, \lambda b, \lambda c, \lambda d, \lambda e, \lambda f$ for λ a nonzero real number?

(ii) Show that the unit circle centered at the origin in $\mathbb{R}^2$ is an example of a real plane conic. What is the collection of $(a, b, c, d, e, f) \in \mathbb{R}^6$ such that the resulting real plane conic is the unit circle centered at the origin in $\mathbb{R}^2$? Which (a, b, c, d, e, f) specify the parabola from part (a)?

(iii) In part (a), three points specified a parabola. How many points in the plane specify a general real plane conic? Why? (Feel free to argue heuristically; we have not yet covered enough linear algebra to formally solve this.)