MATH 201: LINEAR ALGEBRA
HOMEWORK DUE TUESDAY WEEK 13

Problem 1. Let $S = \{(1, 0, i), (1, 2, 1)\}$ in $\mathbb{C}^3$ (with the standard inner product). Compute $S^\perp$.

Problem 2. Let A be an $m \times n$ matrix over F (where $F = \mathbb{R}$ or $\mathbb{C}$). Note that both colspace(A^*) and nullspace(A) are subspaces of F^n. Using the standard inner product on F^n, prove that

$$(\text{colspace}(A^*))^\perp = \text{nullspace}(A).$$

⚠ Beware that $(\ )^*$ denotes conjugate transpose and not duality in this context.

Problem 3. Let V be the vector space of all continuous functions $[0, 1] \to \mathbb{R}$ with inner product $\langle f, g\rangle = \int_0^1 f(t)g(t)\, dt$. Let W be the subspace spanned by $\{t, \sqrt{t}\}$. (*Note*: You can — and should! — check your answers in this problem.)

(a) Apply Gram-Schmidt to $\{t, \sqrt{t}\}$ to compute an orthonormal basis $\{u_1, u_2\}$ for W.

(b) Find the closest function in W to $f(t) = t^2$. Express your solution in two forms: (i) as a linear combination of u_1 and u_2, and (ii) as a linear combination of t and $\sqrt{t}$.