MATH 201: LINEAR ALGEBRA
HOMEWORK DUE FRIDAY WEEK 13

Problem 1. Find the least squares line of best fit to the following points in $\mathbb{R}^2$: $(-1,0),(1,4),(2,4),(3,6)$ via the following methods:

(a) Using Gram-Schmidt to compute an orthogonal projection.
(b) Using the adjoint of a matrix.

Problem 2 (Bonus)**.** Let V be a finite-dimensional inner product space over the field $F = \mathbb{R}$ or $\mathbb{C}$, and let $f \in \mathcal{L}(V)$ be a linear operator on V.

(a) Show that there exists a unique linear operator $f^\dagger \in \mathcal{L}(V)$ with the property that

$$\langle f(x), y\rangle = \langle x, f^\dagger(y)\rangle \quad \text{for all } x, y \in V.$$

(b) Suppose that α is an orthonormal ordered basis for V, and let $A = M_\alpha(f)$ be the matrix representing f with respect to α. Show that $A^\dagger = M_\alpha(f^\dagger)$, where $A^\dagger$ denotes the conjugate transpose of A.
(c) Let A be an $m \times n$ matrix over F. Show that $(Ax)\cdot y = x \cdot (A^\dagger y)$ for all $x \in F^n$ and $y \in F^m$.