

**MATH 113: DISCRETE STRUCTURES  
HOMEWORK DUE FRIDAY WEEK 13**

*Problem 1.* Let  $n$  be a natural number and consider the quantity

$$\psi(n) = \sum_{d|n} \phi(d)$$

which is the sum of the values  $\phi(d)$  where  $d$  ranges through all the positive divisors of  $n$ . What is  $\psi(n)$ ? (Experiment, formulate a conjecture, and prove it. *Hint:* Consider the fractions  $\frac{1}{n}, \frac{2}{n}, \dots, \frac{n}{n}$ .)

*Problem 2.* Use Sunzi's Theorem to efficiently compute the congruence class of  $17^2$  modulo 35. (What is the value to  $\overline{17^2} \in \mathbb{Z}/5\mathbb{Z}$ ? and in  $\mathbb{Z}/7\mathbb{Z}$ ? Use Sunzi's Theorem to push these results back into  $\mathbb{Z}/35\mathbb{Z}$ .)