MATH 113: DISCRETE STRUCTURES
HOMEWORK DUE WEDNESDAY WEEK 10

Problem 1. A women's clinic has four doctors, and each patient is assigned to one of them. If a patient gives birth between 8A.M. and 4P.M., then her chance of being attended by her assigned doctor is $3/4$; otherwise it is $1/4$. What is the probability that a patient is attended by her assigned doctor when she gives birth?

Problem 2. At a certain university, a randomly selected student who has just enrolled has a 66% chance of graduating in four years, but if she successfully completes all freshmen courses in her first year, this chance goes up to 90%. Among those failing to complete at least one freshman course in their first year, the 4-year graduation rate is 50%. What is the percentage of all students who do not complete all freshman courses in their first year?

Problem 3. Let us throw a six-sided die $n-1$ times, and for $1 \le i \le n-1$, denote by A_i the event that throw i results in an even number. Let A_n denote the event that the sum of all the events is even.

(a) Let $\{i_1, i_2, \ldots, i_k\}$ be a k-element subset of $\underline{n}$ where $1 \le k \le n-1$. Prove that

$$P(A_{i_1})P(A_{i_2})\cdots P(A_{i_k}) = \frac{1}{2^k} = P(A_{i_1} \cap A_{i_2} \cap \cdots \cap A_{i_k}).$$

(b) Prove that $P(A_1 \cap A_2 \cap \cdots \cap A_n) = \frac{1}{2^{n-1}}$.

(c) Are the events $A_1, \ldots, A_n$ fully independent?