MATH 113: DISCRETE STRUCTURES
HOMEWORK DUE MONDAY WEEK 10

Problem 1 (*DM:EB* 5.2.3). Consider an experiment with sample space S repeated n times ($n \geq 2$). Let $s \in S$. Let A be the event that the first outcome is s, and let B be the event that the last outcome is s. Prove that A and B are independent (relative to the uniform probability distribution).

Problem 2. Events $A_1, A_2, \ldots, A_k$ are *fully independent* if for all subsets $I \subseteq \{1, 2, \ldots, k\}$,

$$P\Big(\bigcap_{i \in I} A_i\Big) = \prod_{i \in I} P(A_i).$$

(The indexed intersection takes the intersection of all the A_i where i ranges through I; the indexed product takes the product of all the $P(A_i)$ where i ranges through I.) Construct three events A, B, C which are pairwise independent but not fully independent. (*I.e.*, $P(A \cap B) = P(A)P(B)$, $P(A \cap C) = P(A)P(C)$, and $P(B \cap C) = P(B)P(C)$, but $P(A \cap B \cap C) \neq P(A)P(B)P(C)$.) *Bonus*: What is the smallest sample space for this problem?