MATH 113: DISCRETE STRUCTURES
WEDNESDAY WEEK 9 HANDOUT

Problem 1. Show that if A and B are independent, then so are their complements A^c and B^c.

Problem 2. We flip a fair coin n times. Let A be the event that the first coin flip was heads. Let B be the event that the number of heads was even. Let C be the event that the number of heads was more than the number of tails. Which pairs of these three events are independent?

Problem 3. There are n players in a Go tournament. In this problem we will use probability theory to show that for certain n it is possible for every collection of 3 players there exists another player who has beaten them all.

(a) Suppose that the outcome of each game is random. (Perhaps the players are lazy and flip a coin to decide the winner.) Fix a 3-subset $\{x, y, z\}$ of players and some player w not in $\{x, y, z\}$. What is the probability that w wins against x, y, and z? What is the probability that w loses against at least one of x, y, z?

(b) Suppose we have another player w' different from w, x, y, and z. Are the results of w''s matches against x, y, z independent of the results of w's matches?

(c) How many players can appear in the role of w? What is the probability that each of them loses against at least one of x, y, z?

(d) Use your answer to (c) and the fact that there are $\binom{n}{3}$ 3-subsets of $\underline{n}$ to produce an upper bound on the probability that for at least one 3-subset $\{x, y, z\}$, no player beats x, y, and z simultaneously.

(e) What does it mean if your upper bound from (d) is less than 1? Use a computer to determine if there are n for which this happens.