MATH 113: DISCRETE STRUCTURES
MONDAY WEEK 9 HANDOUT

Let S be our sample space (really any set) and let $\mathscr{E} = \mathscr{P}(S)$ denote the corresponding collection of events (just the set of subsets of S). Recall that a *probability distribution* on S is a function

$$P : \mathscr{E} \to [0,1]$$

such that (1) $P(S) = 1$, (2) $P(\varnothing) = 0$, and (3) if A, $B \in \mathscr{E}$ are mutually exclusive events (so $A \cap B = \varnothing$), then $P(A \cup B) = P(A) + P(B)$. If S is a finite set, then we can define the *uniform probability distribution* on S to be the function taking $A \subseteq S$ to $|A|/|S|$.

Problem 1. A lottery has participants choose 5 distinct numbers from the set $\{1, 2, \dots, 36\}$. On a prescribed date, the lottery announces a collection of 5 winning numbers. Complete the following prompts in order to determine why the lottery does not offer a prize for having selected only 1 winning number.

(a) What sample space is pertinent in this question? Describe it both as a collection of certain types of objects, and in a more mathematical fashion.
(b) Is it reasonable to put the uniform probability distribution on this sample space? (Assume that the lottery is fair.)
(c) Let B denote the event of choosing a ticket with no winning numbers. What $P(B)$?
(d) Let A denote the event of choosing a ticket with at least one winning number. What is $A \cap B$? $A \cup B$?
(e) Use the axioms for a probability distribution and your answer to (c) to determine $P(A)$.
(f) [Follow up question] Might it be reasonable to offer prizes for anyone with 2 or more winning numbers?

Problem 2. What is the probability that in a random ordering of a standard deck of cards, the ace of spades precedes the king of hearts?

(a) Rephrase this as a question about permutations of $\underline{52}$. What is the sample space under consideration? the event?
(b) Prove that the probability of this event (under the uniform distribution) is $1/2$ by producing a bijection between the event and its complement. (Why does that solve things?)

Problem 3. Your partner invites you to play a game: s/he writes ten distinct real numbers on ten blank cards. The cards are shuffled randomly and placed face down on the table. You start at the top of the deck and start revealing cards. At any point you may choose to stop turning over cards and select the most recently revealed card. You win if your selection is the largest of all ten numbers (both those previously revealed and those still unrevealed). Devise a strategy which guarantees you will win this game at least 25% of the time.