MATH 113: DISCRETE STRUCTURES
MONDAY WEEK 7 HANDOUT

Problem 1. A *complete graph* on n vertices, denoted K_n, has every possible edge. Draw pictures of K_3, K_4, and K_5. How many edges are there in a complete graph on n vertices? For a general graph $G = (V, E)$, make an inequality relating $|V|$ and $|E|$.

Problem 2. A graph $G = (V, E)$ is called *bipartite* if $V = A \cup B$ with $A \cap B = \varnothing$ and there are no edges between vertices in A and similarly for B (so only edges between a vertex in A and a vertex in B are allowed). The *complete bipartite graph on $p + q$ vertices*, denoted $K_{p,q}$, has $|A| = p$, $|B| = q$, and all possible edges between A and B.

(a) Draw pictures of $K_{2,3}$ and $K_{3,5}$.
(b) How many edges are in $K_{p,q}$?
(c) If $|A| = p$ and $|B| = q$ with $A \cap B = \varnothing$, how many (not necessarily complete) bipartite graphs have vertex set $A \cup B$?

Problem 3. Suppose $G = (V, E)$ and $G' = (V', E')$ are graphs.

(a) When should a function $f : V \to V'$ be considered a "map" $G \to G'$?
(b) When should we consider G and G' to be "the same" graph?