MATH 113: DISCRETE STRUCTURES
WEDNESDAY WEEK 6 HANDOUT

Problem 1 (Continued from Monday). Prove that

$$\binom{n}{0} + \binom{n-1}{1} + \binom{n-2}{2} + \cdots + \binom{0}{n} = F_{n+1}.$$

Problem 2. Extend the Fibonacci sequence backwards (with negative indices) via the relation $F_n = F_{n+2} - F_{n+1}$. Write out the terms $F_{-5}, F_{-4}, F_{-3}, \ldots, F_3, F_4, F_5$ (and maybe a few more in either direction). Come up with a conjecture about the relation between Fibonacci numbers with negative indices and positive indices. Prove your conjecture.

Problem 3. Compute the following sums:

$$F_1$$
$$F_1 + F_3$$
$$F_1 + F_3 + F_5$$
$$F_1 + F_3 + F_5 + F_7$$
$$F_1 + F_3 + F_5 + F_7 + F_9$$

Develop and prove a conjecture about the value of $G_n = \sum_{k=1}^{n} F_{2k-1}$.

Problem 4. Develop and prove a conjecture about the value of $F_{n-1}F_{n+1} - F_n^2$.