

**MATH 113: DISCRETE STRUCTURES**  
**MONDAY WEEK 3 HANDOUT**

*Problem 1.* For the following relations (with their standard meanings), determine what (if any) of the three properties of an equivalence relation they have:  $\neq$ ,  $>$ ,  $\leq$ .

*Problem 2.* Consider the relation  $\sim$  on  $\mathbb{R}$  such that  $x \sim y$  if and only if  $x - y \in \mathbb{Z}$ . Prove that  $\sim$  is an equivalence relation.

*Problem 3.* How many ways can we string  $n$  distinct beads on a necklace? We say that two lists of the  $n$  beads are equivalent if each bead is adjacent to the same two beads on each list. (The first and last beads on the list are considered adjacent.)

- (a) Prove that the above relation on bead lists is an equivalence relation.
- (b) How many lists are in an equivalence class?
- (c) How many equivalence classes are there?

*Problem 4.* Use an equivalence class count to interpret and answer the following question:  $n$  Americans and  $n$  Russians attend a meeting and sit around a round table. If Americans and Russians alternate seats, in how many ways may they be seated?

*Problem 5.* We place two red and two black checkers on the corner of a square. Say that two configurations are equivalent if one can be rotated to the other. Check that this is an equivalence relation, and write down its equivalence classes. Can the number of equivalence classes be found by dividing 6 (the number of words in RRBB) by some natural number?