MATH 113: DISCRETE STRUCTURES
WEDNESDAY WEEK 2 HANDOUT

Problem 1. If $a_1, a_2, \ldots, a_k \in \{0,1\}$, we write $(a_1a_2\ldots a_k)_2$ for the integer represented by this string in base 2; in other words,

$$(a_1a_2\ldots a_k)_2 = a_1 2^{k-1} + a_2 2^{k-2} + \cdots + a_{k-1}2^1 + a_k 2^0.$$

(a) How do you express $2 \cdot (a_1a_2\ldots a_k)_2$ in binary?
(b) Find a closed formula for the n-th term in the sequence 1_2, 11_2, 111_2, 1111_2, $\ldots$.

Problem 2. Suppose A is a nonempty finite set containing n elements and that a is a particular element of A. How many subsets of A contain a? (Try to solve this problem both with a direct count, and also by producing a bijection between $\{B \subseteq A \ : \ a \in B\}$ and a set which you've already counted.)

The second solution method for Problem 1 is an important one in combinatorics. Underlying it is the fact that two sets X and Y have the same cardinality if and only if there is a bijection $X \to Y$. If we know how to count the elements of Y and we can produce a bijection $X \to Y$, then we know X has the same number of elements!

Problem 3. Determine the number of ordered pairs (A, B) where

$$A \subseteq B \subseteq \{1, 2, \ldots, n\}.$$

Problem 4. In what number system can you easily enumerate the above pairs? Use this number system to enumerate such pairs when $n = 3$.

Problem 5. Generalize the above two problem to finite "chains of subsets" $(A_1, A_2, \ldots, A_m)$ where

$$A_1 \subseteq A_2 \subseteq \cdots \subseteq A_m \subseteq \{1, 2, \ldots, n\}.$$