MATH 113: DISCRETE STRUCTURES
HOMEWORK FOR WEDNESDAY WEEK 5

Question 1. How many permutations of an n-element set have exactly one fixed point? Take an integer k such that $1 \leq k \leq n$; how many permutations of an n element set have exactly k fixed points?

Question 2. What is wrong with the following inductive "proof" that $D(n) = (n-1)!$ for all $n \geq 2$?

> For $n = 2$ the formula holds, so take some $n \geq 3$ and assume that $D(n-1) = (n-2)!$. Let π be a permutation of $\{1, 2, \ldots, n-1\}$ with no fixed point. We want to extend it to a permutation π' of $\{1, 2, \ldots, n\}$ with no fixed point. We choose a number $i \in \{1, 2, \ldots, n-1\}$, and we define $\pi'(n) = \pi(i)$, $\pi'(i) = n$, and $\pi'(j) = \pi(j)$ for $j \neq i, n$. This defines a permutation of $\{1, 2, \ldots, n\}$, and it's easy to check that it has no fixed point. For each of the $D(n-1) = (n-2)!$ possible choices of π, the index i can be chosen in $n-1$ ways; therefore, $D(n) = (n-2)! \cdot (n-1) = (n-1)!$.