

**MATH 113: DISCRETE STRUCTURES**  
**FRIDAY WEEK 8 HANDOUT**

*Problem 1.* Show that if  $A$  and  $B$  are independent, then so are their complements  $A^c$  and  $B^c$ .

*Problem 2.* We flip a fair coin  $n$  times. Let  $A$  be the event that the first coin flip was heads. Let  $B$  be the event that the number of heads was even. Let  $C$  be the event that the number of heads was more than the number of tails. Which pairs of these three events are independent?

*Problem 3.* There are  $n$  players in a Go tournament. In this problem we will use probability theory to show that for certain  $n$  it is possible for every collection of 3 players there exists another player who has beaten them all.

- (a) Suppose that the outcome of each game is random. (Perhaps the players are lazy and flip a coin to decide the winner.) Fix a 3-subset  $\{x, y, z\}$  of players and some player  $w$  not in  $\{x, y, z\}$ . What is the probability that  $w$  wins against  $x, y,$  and  $z$ ? What is the probability that  $w$  loses against at least one of  $x, y, z$ ?
- (b) Suppose we have another player  $w'$  different from  $w, x, y,$  and  $z$ . Are the results of  $w'$ 's matches against  $x, y, z$  independent of the results of  $w$ 's matches?
- (c) How many players can appear in the role of  $w$ ? What is the probability that each of them loses against at least one of  $x, y, z$ ?
- (d) Use your answer to (c) and the fact that there are  $\binom{n}{3}$  3-subsets of  $\underline{n}$  to produce an upper bound on the probability that for at least one 3-subset  $\{x, y, z\}$ , no player beats  $x, y,$  and  $z$  simultaneously.
- (e) What does it mean if your upper bound from (d) is less than 1? Use a computer to determine if there are  $n$  for which this happens.