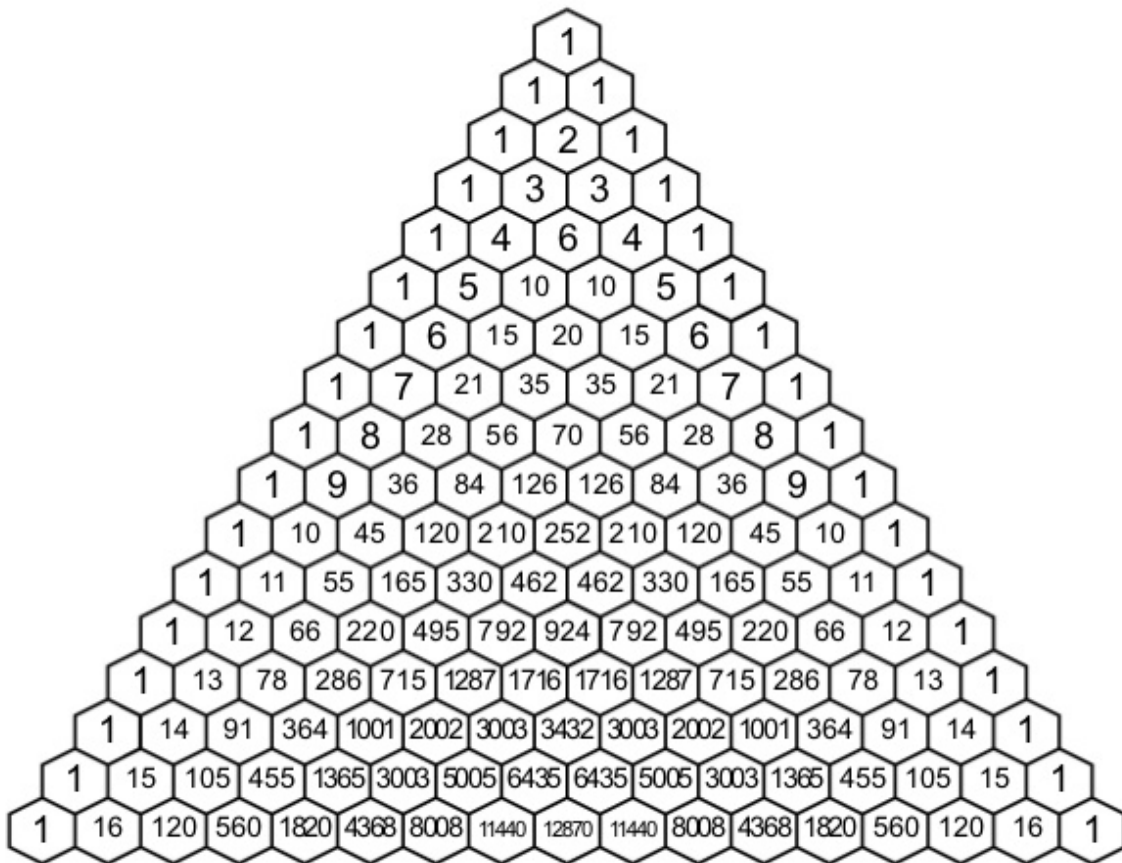


**MATH 113: DISCRETE STRUCTURES**  
**MONDAY WEEK 6 HANDOUT**

*Problem 1.* In how many ways can you fill a  $2 \times n$  chessboard with  $2 \times 1$  dominoes? (Each domino must cover exactly two squares, but may be placed horizontally or vertically.) Work out the answer directly for several small values of  $n$ , make a conjecture about the overall pattern, then prove your conjecture.

*Problem 2.* Mark the first entry in some row of Pascal's triangle (this is a 1). Move one step east and one step northeast, and mark the entry there. Repeat this until you exit the triangle. Compute the sum of the entries you marked.

- (a) Repeat this process for several other rows of Pascal's triangle. Guess what pattern is emerging.  
 (b) Express your guess in terms of a sum of binomial coefficients and prove that it is true.



*Problem 3.* Extend the Fibonacci sequence backwards (with negative indices) via the relation  $F_n = F_{n+2} - F_{n+1}$ . Write out the terms  $F_{-5}, F_{-4}, F_{-3}, \dots, F_3, F_4, F_5$  (and maybe a few more in either direction). Come up with a conjecture about the relation between Fibonacci numbers with negative indices and positive indices. Prove your conjecture.