

**MATH 113: DISCRETE STRUCTURES**  
**WEDNESDAY WEEK 5 HANDOUT**

*Problem 1.* How many derangements  $\pi$  of  $\underline{n}$  have  $\pi(1) = 2$  and  $\pi(2) = 1$ ? Fix some  $k, 2 \leq k \leq n$ ; how many derangements  $\pi$  of  $\underline{n}$  have  $\pi(1) = k$  and  $\pi(k) = 1$ ?

*Problem 2.* How many derangements  $\pi$  of  $\underline{n}$  have  $\pi(1) = 2$  and  $\pi(2) \neq 1$ ? Fix some  $k, 2 \leq k \leq n$ ; how many derangements  $\pi$  of  $\underline{n}$  have  $\pi(1) = k$  and  $\pi(k) \neq 1$ ?

*Problem 3.* Let  $D(n)$  be the number of derangements of  $\underline{n}$ . Use your answers to Problems 1 and 2 to find a formula for  $D(n)$  in terms of  $D(n-2)$  and  $D(n-1)$ . Determine  $D(1)$  and  $D(2)$  by hand and then use your formula to determine  $D(n)$  for  $n = 3, 4, 5$ , and  $6$ ; check that your answers match with the closed formula given by the inclusion-exclusion principle.