MATH 113: DISCRETE STRUCTURES
MONDAY WEEK 5 HANDOUT

The *pigeonhole principle* tells us that if we have n pigeonholes and $k > n$ pigeons, then if we put all the pigeons in pigeonholes, one of the pigeonholes must contain at least two pigeons. In the language of functions, this says that if $f : A \to B$ is a function with $|A| > |B|$, then f is *not* injective. (Careful! It does not say that f is surjective — make sure you appreciate the difference.)

The *generalized pigeonhole principle* says that if there are n pigeonholes and $k > rn$ pigeons where r is a positive integer, then if we put all the pigeons in pigeonholes, one of the pigeonholes must contain at least $r + 1$ pigeons. This is equivalent to the statement that if N objects are put in b boxes, then some box contains at least $\lceil N/b \rceil$ objects.

Problem 1. In a round robin chess tournament with n participants, every player plays every other player exactly once. Prove that at any given time during the tournament, two players have finished the same number of games.

Problem 2. What is the least number of area codes needed to guarantee that the 25 million phones in a state can be given distinct 10-digit telephone numbers of the form NXX-NXX-$XXXX$ where each X is any digit from 0 to 9 and each N represents a digit from 2 to 9? (The area code is the first three digits.)

Problem 3. Show that in the sequence $7, 77, 777, 7777, \ldots$ there is an integer divisible by 2003. (*Hint*: First use "obvious" facts about integer divisibility to prove that if there are terms in the sequence $a_i > a_j$ such that $a_i - a_j$ is divisible by 2003, then there is a term of the sequence divisible by 2003. In order to show that such a_i, a_j exist, note that $a_i - a_j$ is divisible by 2003 if and only if a_i and a_j have the same remainder upon division by 2003; then use the pigeonhole principle.)