

**MATH 113: DISCRETE STRUCTURES**  
**MONDAY WEEK 2 HANDOUT**

The *floor* function  $\lfloor \cdot \rfloor : \mathbb{R} \rightarrow \mathbb{R}$  sends  $x \in \mathbb{R}$  to the greatest integer less than or equal to  $x$ . For instance,  $\lfloor 4.5 \rfloor = 4$ ,  $\lfloor 17 \rfloor = 17$ , and  $\lfloor -\pi \rfloor = -4$ .

*Problem 1.* Draw a graph of  $\lfloor \cdot \rfloor$  and check that it is a function. What is the image of the floor function? Is it injective or surjective?

*Problem 2.* Define  $f : \mathbb{N} \rightarrow \mathbb{Z}$  by

$$f(n) = \begin{cases} \frac{n}{2} & \text{if } n \text{ is even,} \\ \frac{1-n}{2} & \text{if } n \text{ is odd.} \end{cases}$$

Show that  $f$  is a bijection.

*Problem 3.* Suppose  $A$  and  $B$  are finite sets and  $f : A \rightarrow B$  is injective. What can we say about  $|A|$  and  $|B|$ ? What if  $f$  is surjective?

*Problem 4.* Let  $F(A, B)$  denote the set of functions with domain  $A$  and codomain  $B$ . If  $|A|, |B| < \infty$ , what is  $|F(A, B)|$ ? (In other words, how many functions are there with domain  $A$  and codomain  $B$ ?)

Suppose  $A$  and  $B$  are sets and  $f : A \rightarrow B$  is a function. If  $A' \subseteq A$ , then the *image* of  $A'$  in  $B$  is defined as

$$f(A') := \{f(a) \mid a \in A'\}.$$

Note that  $f(A) = \text{im}(f)$ . If  $B' \subseteq B$ , then the *preimage* of  $B'$  in  $A$  is defined as

$$f^{-1}(B') := \{a \in A \mid f(a) \in B'\}.$$

In other words,  $f^{-1}(C)$  consists of everything in  $A$  pushed into  $C$  by  $f$ .

*Problem 5.* Determine  $f(\emptyset)$  and  $f^{-1}(\emptyset)$ . More generally, when is  $f^{-1}(B') = \emptyset$ ?

*Problem 6.* For  $A_1, A_2 \subseteq A$ ,  $B_1, B_2 \subseteq B$ , and  $f : A \rightarrow B$ , prove that

$$f(A_1 \cup A_2) = f(A_1) \cup f(A_2),$$

$$f(A_1 \cap A_2) \subseteq f(A_1) \cap f(A_2),$$

$$f^{-1}(B_1 \cup B_2) = f^{-1}(B_1) \cup f^{-1}(B_2), \text{ and}$$

$$f^{-1}(B_1 \cap B_2) = f^{-1}(B_1) \cap f^{-1}(B_2).$$

Find an example to show that equality does not necessarily hold in the second line.