Math 431 homework Due Friday, September 28, 2001

1. Find a primitive element for the field $\mathbf{Q}(\sqrt{2}, \sqrt[3]{3})$, and find its minimal polynomial.

2. If α is an element of a number field F, prove that there is an integer m such that $m\alpha \in \mathbf{Z}_F$.

3. Prove or disprove: For all positive integers n, $\sin(\pi/n)$ is an algebraic number. Prove or disprove: For all positive integers—n, $\sin(\pi/n)$ is an algebraic integer.

4. Find all units in the ring of integers in $\mathbf{Q}(\sqrt{14})$.

5. If $\mathrm{m}_\alpha(x) = x^2 + ax + b$ over a field F, find $\mathrm{disc}(\alpha)$ in $F(\alpha)$. Same question if $\mathrm{m}_\alpha(x) = x^3 + ax + b$.

6. Show that if $a \in \mathbf{Z}$ and $b \in \mathbf{Z}$ can be written as the sum of two squares then so can ab. Show that if $a \in \mathbf{Z}$ and $b \in \mathbf{Z}$ can be written in the form $x^2 + xy - y^2$ for integers x, y, then so can ab.

7. (*) I had intended to put the following problem (due to Iraj Kalantari) on the first homework assignment as a review problem. But forgot. So perhaps you will be willing to indulge me, and consider it now.

10201 people lived in Sherwood Forest. Everyone had a mate except the priest. Each person had written a song. Singing was contagious.

Every song was sung to every person by a singer. (This includes the fact that each person's own song was sung to that person by some singer.) The priest, to whom everyone sang one's song, had a song that everyone sang to oneself. The priest sang to each person the song written by that person's mate and, having no mate, he sang his own song to himself.

Any singer who sang to a first person the song of a singer of a second person's song to a third, was the same singer who sang to the singer of the third person's song to the first person the song of the second person.

If it was Marian who sang Robin's song to Little John, who sang Little John's song to Robin? Who sang Marian's song to her?