Math 431 homework Due Friday, September 14, 2001

1. Find the product of all nonzero elements of a finite field with q elements.

2. If E/F is a finite extension of fields, prove that every element of E is algebraic over F.

3. Prove or disprove: the field $\mathbf{Q}(\sqrt{5})$ is isomorphic to the field $\mathbf{Q}(\sqrt{7})$.

4. Prove that if E is a splitting field of a polynomial $f(x)$ of degree n over a field F, then $[E : F] \leq n!$.

5. Let F be a root field over $\mathbf{Q}$ of the polynomial $f(x) = x^4 - 2$. Factor $f(x)$ in $F[x]$.

6. Same as the previous question, except with $f(x) = x^4 + 1$.

7. Find a field F inside $\mathbf{R}$ of as small a degree over $\mathbf{Q}$ as possible such that there is a regular pentagon in the plane whose vertices (x, y) have their coordinates x, y lying in F.

8. Let a and b be integers and p a prime. Show that

$$\binom{pa}{pb} \equiv \binom{a}{b} \bmod p.$$