Math 431 homework Due Friday, November 16, 2001

1. Find integers x and y such that

$$2x^2 + 2xy + 3y^2 = 1607.$$

Explain your methods. Brute force techniques are acceptable if you replace the RHS by $1607 \cdot (10^{50} + 1)$. (Numerical hint: in building the norm table, it turns out that the prime 1607 first appears in row 363.)

2. Show that if $f(x) \in \mathbf{Z}[x]$ is "Eisenstein" for a prime p then p does not divide the index $[\mathbf{Z}_F : \mathbf{Z}[\alpha]]$, where α is a root of f, and $F = \mathbf{Q}(\alpha)$.

3. Let $F = \mathbf{Q}(\sqrt[3]{m})$, where m is not a perfect cube. Show that if p is a prime, not equal to 3, such that p divides m but p^2 does not divide m, then p does not divide the index $[\mathbf{Z}_F : \mathbf{Z}[\alpha]]$. Show that if 3 does divide this index then $m \equiv 0 \bmod 9$ or $m \equiv \pm 1 \bmod 9$. Is it possible for 3 to be unramified in F for a suitable m? Find the ring of integers in $\mathbf{Q}(\sqrt[3]{289})$.

4. Find a nontrivial unit in, and the class number of, $\mathbf{Q}(\sqrt[3]{22})$.

5. Find the ring of integers in $\mathbf{Q}(\sqrt{5}, \sqrt{13})$.