DEFICIENCIES IN THE DIRICHLET ARITHMETIC PROGRESSION WRITEUP

- *Insufficient analytic detail about Dirichlet series.* The pending writeup on the class number formula for imaginary quadratic fields is more thorough in this context.
- *Ad hoc continuation of* $\zeta(s)$ *to* $0 < \mathrm{Re}(s) < 1$. The auxiliary writeup on continuations and functional equations for $\zeta(s)$ and $L(s, \chi)$ addresses this point, and the next one. The auxiliary writeup on local factors of zeta functions is also relevant.
- *Conditional convergence of* $L(s, \chi)$ *on* $0 < \mathrm{Re}(s) < 1$. See the comment in the previous bullet.
- *Unmotivated, fragile argument that* $L(1, \chi) \neq 0$. The auxiliary writeup on the cyclotomic zeta function partly addresses the motivation, but not the fragility. The writeup on a quadratic zeta function discusses some related ideas—both zeta functions are examples of Dedekind zeta functions, to play a key role in the class number formula for imaginary quadratic fields. The writeup on $L(1, \chi)$ is less fragile but again unmotivated. A motivated, robust treatment of the issue is beyond our scope.
- *What to do next?* The writeup about Gaussian primes in sectors sketches the beginning of a vast generalization of Dirichlet's Theorem on primes in arithmetic progressions.