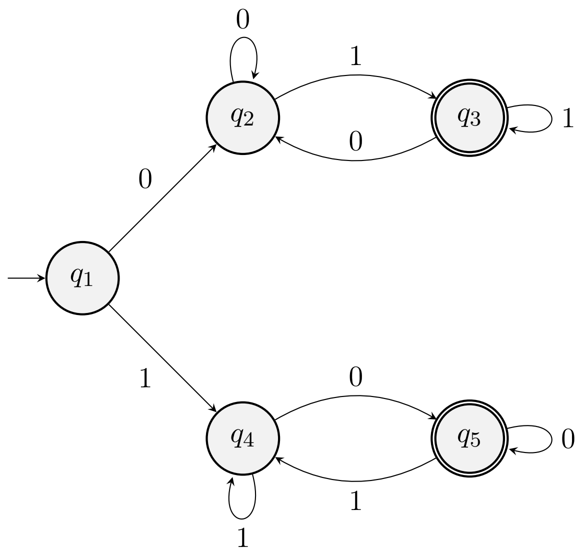
PROBLEM 1. Give two branching programs B_1 and B_2 that agree on all Boolean input but have different output polynomials when arithmetized. (Try to find a simple example.)

PROBLEM 2.

(a) Prove that if A is a regular language, a family of branching programs $(B_1, B_2, \ldots)$ exists where each B_n accepts exactly the strings in A of length n and is bounded in size by a constant times n. It may help to describe your BP one level at a time. (Take the size of a BP to be the number of nodes plus the number of edges.)
   (i) Describe the construction of each B_n.
   (ii) Prove that the size of B_n is $O(n)$.

(b) Illustrate your proof by creating B_3 for the regular language generated by the following DFA:



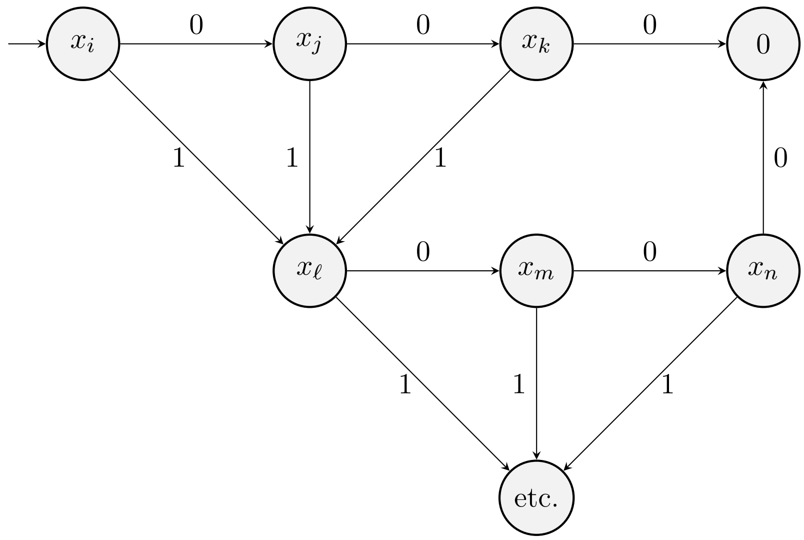
PROBLEM 3. Let

$$\text{ISO} = \{\langle G, H\rangle : G \text{ and } H \text{ are isomorphic graphs}\}.$$

Explain why ISO $\in$ IP.

PROBLEM 4. The *parity function on n inputs* is the Boolean function that returns 1 if the number of 1s in the input is odd, and returns 0, otherwise. Show that the parity function with n inputs can be computed by a branching program that has $O(n)$ nodes by drawing the example when $n = 4$ in a way that clearly generalizes to arbitrary n.

PROBLEM 5. Prove that EQBP is coNP-complete. Here is a hint due to Moss for converting a Boolean formula in 3CNF form into a BP in polynomial time:



x_i
x_j
x_k
0
x_ℓ
x_m
x_n
etc.
0
0
0
1
1
1
0
0
0
1
1
1