PROBLEM 1. Prove that $\text{SPACE}(n^k) \subsetneq \text{SPACE}(n^{\log(n)})$.

PROBLEM 2. Show that if $\text{NP} = \text{P}^{\text{SAT}}$, then $\text{NP} = \text{coNP}$.

PROBLEM 3. Is $\text{NTIME}\,(n^2) \subsetneq \text{PSPACE}$? Explain.

PROBLEM 4. To prove that $\text{P}^{\text{TQBF}} = \text{NP}^{\text{TQBF}}$ we use the following flag of complexity classes
$$\text{NP}^{\text{TQBF}} \subseteq \text{NPSPACE} = \text{PSPACE} \subseteq \text{P}^{\text{TQBF}}.$$
Justify each step.

PROBLEM 5.

(a) Construct AND and OR gates using BPs.

(b) Consider the arithmetized versions of the BP you created for AND and OR gates. Let the label a of the starting node be 1, as in Sipser's lecture. Considering the Boolean variables x_1 and x_2 as indeterminates, what is the output polynomial of each of your BPs?

(c) Check that each polynomial gives the right values when the variables are assigned bit values.

PROBLEM 6. Show that $\text{BPP} \subseteq \text{PSPACE}$.