CS/Math 387 Homework for Friday, Week 11

PROBLEM 1. Review.

(a) If $A \leq_L B$ and B is in L, why is A in L? (How does one get around the problem that if f is the mapping reduction, $f(w)$ my have length that is longer than $O(\log(n))$?

(b) Why is NL is in P?

PROBLEM 2. Let A be the language of properly-nested parentheses. For example, A contains (()) and ()(()()) but not ())(. Show that A is in L.

PROBLEM 3. A directed graph is *strongly connected* if for all ordered pairs of vertices (u, v) there is a directed path from u to v. Thus, there is a directed path between every pair of vertices in both directions. Let

$$SC = \{\langle G \rangle : G \text{ is a strongly connected directed graph}\}.$$

(a) Show that SC is in NL.

(b) Given a directed graph G and two vertices s and t of G. Create a new graph G' with the same vertices as G such that G' is strongly connected if and only if there is a path in G from s to t.

(c) Show that SC is NL-complete by reducing from PATH.

PROBLEM 4. Review.

(a) State the space hierarchy theorem.

(b) For any two rational numbers $0 \leq a < b$, show that $n^a \in o(n^b)$ and hence, via the space hierarchy theorem, $\text{SPACE}(n^a) \subsetneq \text{SPACE}(n^b)$. Show the relevant calculation. (Here, we are using the fact that n^c is space constructible for any rational number c.)

(c) Savitch's theorem says $\text{NL} \subseteq \text{SPACE}(\log^2(n))$. Use this to prove that $\text{NL} \subsetneq \text{PSPACE}$. Show the relevant calculation.

PROBLEM 5.

(a) Prove that $\text{TIME}(2^n) = \text{TIME}(2^{n+1})$.

(b) Prove that $\text{TIME}(2^n) \subsetneq \text{TIME}(2^{2n})$.

PROBLEM 6.

(a) Explain why A_{NFA} is in NL.

(b) Prove that A_{NFA} is NL-complete by giving a reduction from PATH.