

PROBLEM 1. Suppose that a TM runs in $O(n^2)$ space? Is there an upper bound on its time complexity? Explain.

PROBLEM 2.

- (a) Carefully explain why $NP \subseteq PSPACE$.
- (b) Suppose that A is PSPACE-hard. Explain why it follows that A is NP-hard. (To say A is PSPACE-hard means that any language in PSPACE can be reduced to A in polynomial time. We do not require that A , itself, is in PSPACE. Similarly, we say A is NP-hard if every language in NP has a polynomial-time reduction to A .)
- (c) Suppose that every NP-hard language is PSPACE-hard. Would that imply $NP = PSPACE$? Explain.

PROBLEM 3.

- (a) Encode the formula

$$\exists x_1 \forall x_2 \exists x_3 [(x_1 \vee x_2) \wedge (x_2 \vee x_3) \wedge (\overline{x_2} \vee \overline{x_3})]$$

as a game of generalized geography using the method described in our book. The players are Player \exists and Player \forall . Player \exists starts at the first node and gets to decide the next node.

- (b) State a winning strategy for one of the players in the geography game, and describe how this strategy gives the truth value of the formula.

PROBLEM 4.

- (a) Perform the following multiplication problem of two binary numbers using the standard algorithm from grade school:

$$\begin{array}{r} 101101 \\ \times 1011 \\ \hline \end{array}$$

This exercise will help you think about the next part of this problem.

- (b) Let

$$\text{MULT} = \{a\#b\#c : a, b, c \text{ are binary natural numbers and } a \times b = c\}.$$

Show that $\text{MULT} \in L$. [Hints: Storing the whole table of values created by standard multiplication cannot be done in log space. However, you are allowed to use a fixed number of pointers to the read-only tape and an additional $O(\log(n))$ space on the work tape (why?). Storing the carry might be a worry, but what is the maximum size of the carry compared to the length of a and b ?]

PROBLEM 5. The cat-and-mouse game is played by two players, “Cat” and “Mouse”, on an arbitrary undirected graph. At a given point, each player occupies a node of the graph. The players take turns moving to a node adjacent to the one that they currently occupy. A special node of the graph is called “Hole”. Cat wins if the two players ever occupy the

same node. Mouse wins if it reaches the Hole before the preceding happens. The game is a draw if a situation repeats (i.e., the two players simultaneously occupy positions that they simultaneously occupied previously, and it is the same player's turn to move). Define

HAPPY-CAT = $\{\langle G, c, m, h \rangle \mid G, c, m, h \text{ are respectively a graph, and positions}$
of the Cat, Mouse, and Hole, such that Cat
has a winning strategy if Cat moves first $\}$.

Show that HAPPY-CAT is in P. (Hint: The solution is not complicated and doesn't depend on subtle details in the way the game is defined. Consider the entire game tree. It is exponentially big, but you can search it in polynomial time. To get a feel for the solution, start by listing several instant winning positions. Positions one away from a winning position?)