CS/Math 387 Homework for Friday, Week 6

PROBLEM 1.

(a) What is a *computable function*? (Give a precise definition that includes stating the domain and codomain.).

(b) Let A, B be languages. What does it mean to say A is *reducible* to B? What does it mean to say A is *mapping reducible* to B?

(c) Why is A always reducible to $\overline{A}$?

(d) Give an example, with explanation, of a language that is reducible to its complement but not mapping reducible to its complement.

PROBLEM 2. Show that a language A is recognizable if and only if $A \leq_m A_{TM}$. (Divide your proof into two parts. In the first, assume that A is recognizable and provide an explicit mapping reduction. This will be a computable function $f\colon \Sigma^* \to \Sigma^*$. So f must account for every possible string, and its output must be a string.)

PROBLEM 3. Let $T = \{\langle M\rangle : M \text{ is a TM and some word } w \in L(M) \text{ has even length}\}$.

(a) Use Rice's theorem to prove that T is undecidable. (Explicitly state the property and show it is non-trivial.)

(b) Show T is undecidable by giving a reduction of A_{TM} to T.

PROBLEM 4. (bonus) Find a match in the following instance of the Post Correspondence Problem:

$$\left\{ \left[\frac{aa}{abab}\right], \left[\frac{b}{a}\right] \left[\frac{aba}{b}\right] \left[\frac{aa}{a}\right] \right\}.$$