CS/Math 387 Homework for Friday, Week 5

Reminder. Here is a reminder of the academic honesty policy with regard to homework in CS/Math 387. Under no circumstances should you search for solutions to homework problems on the internet—that includes the bonus problems. You are permitted and encouraged to work with your peers on homework problems. However, you must write up your solutions independently and you should not turn in solutions you do not understand. For instance, sharing your written solutions or receiving written solutions from others is a violation of this policy. It is best practice to cite those with whom you worked *outside of group work*.

PROBLEM 1. Let $A = \{\langle R, S\rangle : R \text{ and } S \text{ are regular expressions and } L(R) \subseteq L(S)\}$. Show that A is decidable.

PROBLEM 2. A *useless state* in a PDA is a state that is never entered on any input string. Let $L = \{< P >: P \text{ is a PDA with a useless state}\}$. Prove that L is decidable.

PROBLEM 3. Prove that if G is a CFG is in Chomsky normal form, then for any string w of length $|w| = n \geq 1$, exactly $2n - 1$ steps are required for any derivation of w. (For the relevance of this problem, recall that our text uses it to prove that the acceptance question for CFGe is decidable, i.e. the language $A_{\text{CFG}} = \{\langle G, w\rangle : G \text{ is a CFG that generates } w\}$ is decidable.) Your proof will judged according to the clarity of its exposition—it should not require too much ingenuity on the part of the reader.

PROBLEM 4. Let

$$E_{\text{TM}} = \{\langle M\rangle : \ M \text{ is a TM and } L(M) = \emptyset\}.$$

Prove that E_{TM} is co-Turing-recognizable.

PROBLEM 5. Let $\Sigma = \{0, 1\}$. Determine, with explanation, which of the following are countable:

(a) The set of all languages, Σ^*.

(b) The set of all strings Σ^*.

(c) Is every regular language countable?

(d) Is the set of all regular languages countable?

PROBLEM 6. Let S be a set, and let $\mathcal{P}(S)$ be the set of all subsets of S.

(a) Given an explicit injective function $S \to \mathcal{P}(S)$.

(b) Show there is no surjective function $S \to \mathcal{P}(S)$. (Hint: Give a proof by contradiction. Suppose you have a surjective function $f\colon S \to \mathcal{P}(S)$. The following set may be helpful: $X = \{s \in S : s \notin f(s)\}$.)

PROBLEM 7. (bonus)

(a) Let

$$E = \{\langle M \rangle : M \text{ is a DFA that accepts some string with more 1s than 0s}\}.$$

Is E decidable?

(b) Write a program (in any programming language) that prints its own code to the screen. The program cannot interact with the file that contains the program. (It can't, for example, read or copy the file that contains its code.)