CS/Math 387 Homework for Friday, Week 4

PROBLEM 1.

(a) Discuss how you would prove that the intersection of a context-free language and a regular language is a context-free language. Afterwards, give a written explanation using fewer than three sentences.

(b) Explain how to prove that the set of binary strings with equal numbers of 0s and 1s but containing no substring of the form 0100 or 1001 is context free. Detailed descriptions of machines are not required.

PROBLEM 2. Let $\Sigma = \{1, 2, 3, 4\}$ and L be the set of words $w \in \Sigma^*$ such that the number of 1s in w equals the number of 2s in w, and the number of 3s in w equals the number of 4s. Prove that L is not a CFL.

PROBLEM 3. For this problem, refer to the formal definition of a Turing machine in the text. Make sure to read the fine print that occurs in the paragraph after the definition.

(a) Can a Turing machine ever write the blank symbol ␣on its tape?

(b) Can the tape alphabet Γ ever be the same as the input alphabet Σ?

(c) After executing a transition, is it ever possible that the head of a Turing machine is over the same cell it started in?

(d) Can a Turing machine contain just a single state?

PROBLEM 4. Review the concept of an enumerator. Let $\Sigma = \{0, 1\}$. *Standard string order* on Σ^* means ordering strings in increasing length and ordering strings of the same length in *lexicographic order* taking $0 < 1$:

$$\varepsilon, 0, 1, 00, 01, 10, 11, 000, 001, 010, 011, 100, 101, 110, 111, 0000, \ldots$$

Show that a language L is Turing decidable if and only if some enumerator enumerates the language in standard string order. (Note: this is an "if and only if" proof. So you need to clearly divide you proof into to parts to prove both direcations.)

PROBLEM 5.

(a) Draw the state machine for a TM that takes as input a word of the form $w\#v$ where $w, v \in \{0, 1\}^*$, and outputs $w\text{␣}\#v$. Notate your transitions as in the text. For example,

$$q_1 \xrightarrow{0 \to 1, R} q_2$$

means that if the machine is in state q_1 and the head reads a 0, then it writes a 1, moves the head to the right, and moves to state q_2. There is also a shorthand notation,

$$q_1 \xrightarrow{0 \to R} q_2$$

means that if the machine is in state q_1 and the head reads a 0, then it writes a 0, moves the head to the right, and moves to state q_2.

(b) Trace the set of configurations your TM moves through starting with the input 001#10. Use the notation from the book, e.g., $11q_2 01$ means machine is in state q_2 and the head is reading a 0.

PROBLEM 6. Prove that the class of context-free languages is closed under reversal using CFGs (not PDAs). (The *reversal* of a language A is the language $\{w^R : w \in A\}$. (Chomsky normal form might help.)

PROBLEM 7. (Bonus) Consider a Turing machine that cannot write over its input. That is, whatever length of string that the input is on cannot be changed, but can be read as normal (and the rest of the tape can be changed as normal). Show that Turing machines of this type can recognize only regular languages.