CS/Math 387 Homework for Wednesday, Week 3

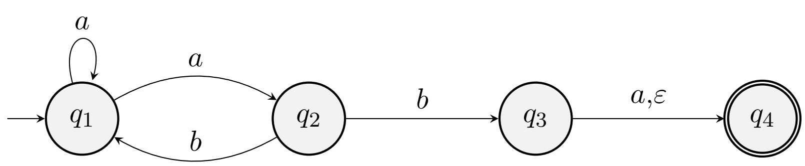
Problem 1. For each of the following, prove either that the language is regular or that it is not regular. In all cases $\Sigma = \{0.1\}$.

(a) $L_1 = \{w : w \text{ contains an equal number of 0s and 1s}\}$.

(b) $L_2 = \{1^k w : w \in \Sigma^*, k \geq 1, \text{ the number of 1s in } w \text{ is at least } k\}$.

(c) $L_3 = \{1^k w : w \in \Sigma^*, k \geq 1, \text{ the number of 1s in } w \text{ is at most } k\}$.

Problem 2. Let A be the NFA pictured below:



(a) Create a CFG G such that $L(A) = L(G)$ *using the algorithm described in Section 2.1 of our text.* (See the part of Section 2.1 entitled "Designing Context Free Grammars".)

(b) Give a derivation of the word *aabab* making one substitution at each step.

Problem 3. Let $P = \{0^q : q \text{ is a prime number}\}$. Prove or disprove that P is regular.

Problem 4. Let $\Sigma = \{0, 1\}$, and consider the language $L = \{0^m 1^n : m \neq n\}$.

(a) Find a CFG generating L using at most three variables.

(b) Convert your CFG into a PDA using the algorithm in the text (as in Problem 1).

(c) Is L regular? Prove or disprove.

Problem 5.

(a) Show that context free languages are closed under the union, concatenation, and star operations. (By the way: they are not closed under intersection and complementation, in general.) For notation, let A and B be context free grammars with start states S_A and S_B, respectively. Describe context free grammars generating the languages $L(A) \cup L(B)$, $L(A)L(B)$, and $L(A)^*$. You may assume that the variables for A and B are distinct.

(b) With notation as above, give an example that shows $L(A)^*$ is not necessarily generated by the context free grammar obtained from A by adding the rule $S_A \to S_A S_A$ to the starting rules for A.

Problem 6. Convert the following CFG into Chomsky normal form using the procedure given in Theorem 2.9:

$$S \to SSX | X0 | \varepsilon$$
$$X \to 10 | \varepsilon.$$

The procedure in Theorem 2.9 has four main stages. Divide your write-up into four clearly marked corresponding parts.

PROBLEM 7. (Bonus problems)

(a) Our goal in this problem is to show that the representation of objects can affect whether or not a given set can be recognized by a machine. Consider a set A of natural numbers. Let $B_k(A)$ be the set of strings that represent numbers from A in base k (with no leading zeros). For example, if $A = \{3, 5\}$ then $B_2(A) = \{11, 101\}$ and $B_3(A) = \{10, 12\}$. We can think of $B_k(A)$ as a language with a k-symbol alphabet. Give a set A for which $B_2(A)$ is regular but $B_3(A)$ is not (and prove it).

(b) Create a CFG that generates the language $L = \{xy : |x| = |y|, x \neq y\}$.