

PROBLEM 1. Here is the Euclidean algorithm for finding the greatest common divisor of two integers:

Input: $(u, v) \in \mathbb{Z}^2$.

```
def EUCLID( $u, v$ ):  
    If  $v = 0$ , then  
        return  $u$   
    else  
        return EUCLID( $v, u \% v$ ).
```

Here, $u \% v$ denotes the remainder of u upon division by v .

Find the lexicographically smallest input (u, v) with $u > v \geq 0$ such that the Euclidean algorithm takes n division steps. Start with the cases $n = 0, 1, 2, 3, 4$.

REVIEW

PROBLEM 2. Is P closed under concatenation? Prove by describing a decider, or give a counterexample.

PROBLEM 3. Is P closed under the Kleene star operator? Prove by describing a decider, or give a counterexample.

PROBLEM 4. What is the statement of Savitch's theorem?

PROBLEM 5. Is $\text{NP} \subseteq \text{PSPACE}$?

PROBLEM 6.

- (a) What is the complexity class L ? NL ?
- (b) Prove that $NL \subsetneq \text{PSPACE}$.

PROBLEM 7. Prove that $\text{NP} \neq \text{TIME}(n)$. Generalize this result.

PROBLEM 8. What is the Church-Turing thesis?

PROBLEM 9. Find a match in the following instance of the Post Correspondence Problem:

$$\left\{ \left[\frac{a}{baa} \right], \left[\frac{ab}{aa} \right], \left[\frac{bba}{bb} \right] \right\}.$$