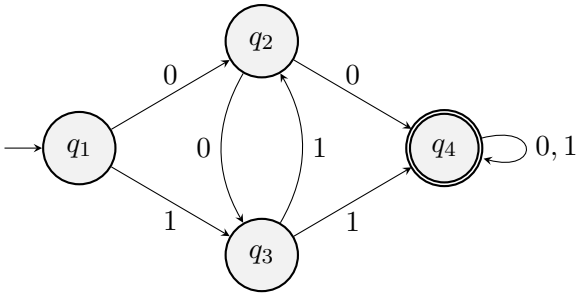
Math 387 Group problems, Monday Week 13

REVIEW (Several of these problems are variations of problems I found online for a course at York University.)

PROBLEM 1. Give a regular expression that generates the same language as the FA shown below:



PROBLEM 2. Give an example of two recognizable languages L_1 and L_2 such that $L_1 \setminus L_2$ is not recognizable. Explain your answer.

PROBLEM 3. Prove whether each of the following languages is decidable:

(a)

(i) $L_1 = \{\langle M \rangle : M \text{ is a TM that accepts every string}\}$.

(ii) $L_2 = \{\langle M \rangle : M \text{ is a TM that rejects some string}\}$.

(iii) $L_3 = \{\langle M \rangle : M \text{ is a TM that } L(M) \text{ is a CGL}\}$.

(iv) $L_4 = \{\langle M \rangle : M \text{ is a TM that accepts some string after running at most 10 steps}\}$.

(b) State Rice's theorem.

(c) For each of the above, explain why Rice's theorem above or why it does not.

PROBLEM 4. Consider the language

$$L = \{\langle M \rangle : M \text{ is a Turing machine which upon input } \varepsilon \text{ eventually prints a character on its 1000-th tape cell}\}.$$

What is the status of L: decidable, undecidable, recognizable, unrecognizable?

PROBLEM 5. You would like to prove that a language L is recognizable but not decidable. You could do this by proving two of the following. Which two?

(a) $L \leq_m A_{TM}$

(b) $L \leq_m \overline{A_{TM}}$

(c) $\overline{L}$ is decidable

(d) $\overline{A_{TM}} \leq_m L$.

(e) $\overline{E_{TM}} \leq_m L$

PROBLEM 6. Is the following language decidable? Recognizable? Provide a proof.

$$\{\langle G_1, G_2\rangle : G_1 \text{ and } G_2 \text{ are CFGs and } L(G_1) \neq L(G_2)\}.$$

PROBLEM 7. Give an example of a non-regular language A such that A^* is regular.