PROBLEM 1. Review. What does it mean for a language A to be in IP? Review the conditions for when $w \in A$ and $w \notin A$.

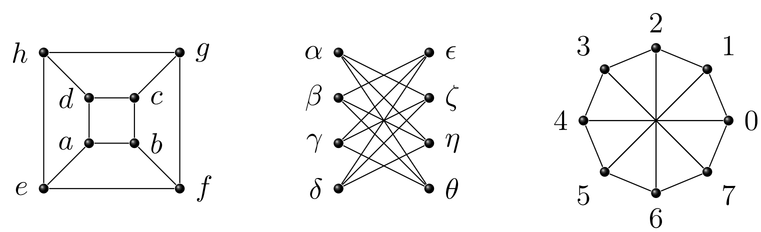
PROBLEM 2. Explain why NP $\subseteq$ IP. Is it the case that NP = IP?

PROBLEM 3. Let

$$\text{ISO} = \{\langle G, H\rangle : G \text{ and } H \text{ are isomorphic graphs}\}.$$

Explain why ISO $\in$ IP.

PROBLEM 4. Determine which pairs of the following are isomorphic, if any.

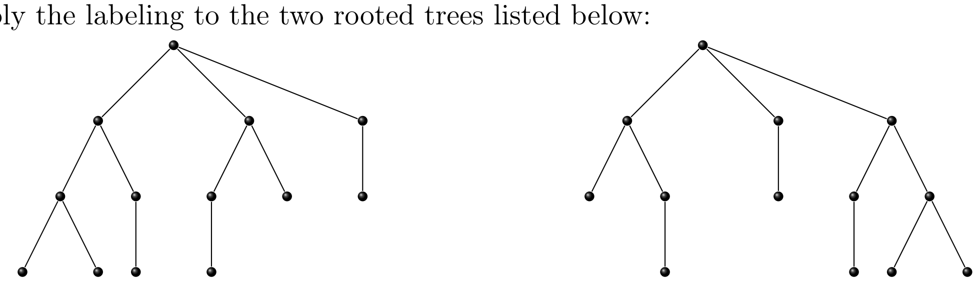


PROBLEM 5. Here we describe an efficient algorithm for determining whether two trees are isomorphic. We start with a labeling system for rooted trees, i.e., pairs (T, r) where T is a tree and r is a fixed vertex of T. We recursively assign a label to T consisting of a balanced parenthetical expression as follows:

1. Leaf nodes are assigned the label ().
2. Every time we move up a level (in towards the root), label the vertex by listing the labels of its subtrees, sorted lexicographically by increasing size, and surround the resulting sorted list by ().
3. A vertex cannot be labeled until all of its children are labeled.

It turns out that two rooted trees are isomorphic if and only if there roots have the same labels.

(a) Apply the labeling to the two rooted trees listed below:



(b) What is the runtime of this algorithm, in general?

(c) For the case of unrooted trees, we first define the *center* of a tree. The center is a either a vertex or a pair of vertices found by first coloring all of the leaf vertices. The uncolored vertices form a tree. Color its vertices. Repeat until there are at most two vertices remaining. Find the center of the tree on the left in the previous part of this problem.

(d) Describe how to use the idea of a center of a tree to solve the isomorphism problem for trees (unrooted) in polynomial time.

PROBLEM 6. (bonus) Fix a set of k colors, $C = \{0, 1, \ldots, k-1\}$. A *$C$-colored graph* is a finite directed simple graph G in which each vertex is assigned a color in C. Two such graphs are isomorphic if there is a graph isomorphism between them (i.e., a bijection ϕ of the vertices inducing a bijection of the edges) that preserves vertex color (i.e., if a vertex v has color j, then so does vertex $\phi(v)$). Consider the language

$$\text{COL-ISO} = \{\langle G, H\rangle : G \text{ and } H \text{ are isomorphic } C\text{-colored graphs}\}.$$

Show that

$$\text{COL-ISO} \leq_P \text{ISO},$$

where ISO is the language corresponding to the usual (non-colored) graph isomorphism problem.