Math 387 Group problems, Friday Week 12

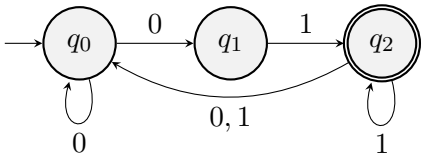
Review

Problem 1. Create a DFA for the language

$$A = \{w : w \text{ is a binary representation of a natural number that is a multiple of } 4\}.$$

We take *binary representation* to mean that w will have leftmost digit equal to 1 unless $w = 0$.

Problem 2. Convert the following NFA into a DFA using the subset construction. Use only the nodes that are necessary.



Problem 3. Consider the language

$$E_{\text{CFG}} = \{\langle G\rangle : G \text{ is a CFG and } L(G) = \emptyset\}.$$

Prove that E_{CFG} is decidable.

Problem 4. Is the subset of a context free language necessarily context free? Give a proof or a counter-example.

Problem 5. Consider the language

$$B = \{\langle M\rangle : M \text{ is a DFA that accepts at least one (nonempty) palindrome}\}.$$

Prove that B is decidable. (Hint: context free languages are not closed under intersection, but ...)

Problem 6. Let $\Sigma = \{a, b, c\}$, and consider the following language:

$$L = \{a^m b^n c^r : m \leq n \text{ and } m, n, r \geq 0\}.$$

Is L regular? Prove your answer is correct.

Problem 7. Let M be a Turing machine. Is it possible that $L(M) = \{\varepsilon\}$? If so, is it possible for ε to be accepted with the TM taking no steps?

Problem 8. Let $E_{\text{TM}} = \{\langle M\rangle : M \text{ is a TM and } L(M) = \emptyset\}$. Is $\overline{E_{\text{TM}}}$, the complement of E_{TM}, Turing-recognizable? Prove your answer.