

PROBLEM 1. Review.

- (a) What is a probabilistic TM? What is the probability of a branch in a probabilistic TM? What is the probability that a word is accepted?
- (b) Let M be a PTM. What does it mean to say that M decides a language A with error probability ε (where $0 \leq \varepsilon < \frac{1}{2}$)?
- (c) What is the class BPP?

PROBLEM 2.

- (a) Create a BP that computes the Boolean function given by the table

n	$f(n)$
00	1
10	1
01	0
11	0.

- (b) Consider the arithmetized version of the BP you created in the previous part of this problem. Let the label a of the starting node be 1, as in Sipser's lecture. Considering the Boolean variables x_1 and x_2 as indeterminates, what is the output polynomial of your BP? (Check that it gives the right values when the variables are assigned bit values.)
- (c) Construct AND and OR gates using BPs.
- (d) Consider the arithmetized version of your AND BP letting the Boolean variables be indeterminates. What is the output polynomial? (Check your solution has the right values when the variables are assigned bit values.)

PROBLEM 3. Show that $\text{BPP} \subseteq \text{PSPACE}$.

PROBLEM 4. [NOTE: This problem is too easy. Next time randomize the values of f .]
Create a BP that computes the boolean function given by the table

n	$f(n)$
000	1
001	1
010	1
011	1
100	0
101	0
110	0
111	0

PROBLEM 5. (bonus) Describe how to construct a BP that computes any given Boolean function $f: \{0, 1\}^m \rightarrow \{0, 1\}$.

PROBLEM 6. (bonus) If $EQBP \in BPP$, show that $NP \subseteq BPP$.

PROBLEM 7. (bonus) Show that $EQBP$ is coNP-complete:

- (a) Why is $EQBP$ in coNP?
- (b) Given any language A in coNP, give a polynomial time mapping to EQBP.