PROBLEM 1. Prove that if $A \in \mathrm{P}$, then $\mathrm{P}^A = \mathrm{P}$. (Argue both containments.)

PROBLEM 2. Show that if $\mathrm{NP} = \mathrm{P}^{\mathrm{SAT}}$, then $\mathrm{NP} = \mathrm{coNP}$.

PROBLEM 3. Review

(a) What is the definition of EXPSPACE?

(b) Prove that for each positive integer k, we have

$$\mathrm{SPACE}(n^k) \subsetneq \mathrm{SPACE}(n^{\log(n)}) \subsetneq \mathrm{SPACE}(2^n).$$

Back up your argument with the relevant calculations.

(c) Why can't an EXPSPACE-complete language be decided in polynomial time?

(d) Is NTIME $(n^2) \subsetneq$ PSPACE? Explain.

(e) Consider this flag of complexity classes:

$$\mathrm{L} \subseteq \mathrm{NL} \subseteq \mathrm{P} \subseteq \mathrm{NP} \subseteq \mathrm{PSPACE} \subseteq \mathrm{EXPTIME} \subseteq \mathrm{EXPSPACE}.$$

Applying the space and time hierarchy theorems, there are three main pairs of classes we can separate. Which are these? Explain.

(f) What completeness property does TQBF have?

(g) To prove that $\mathrm{P}^{\mathrm{TQBF}} = \mathrm{NP}^{\mathrm{TQBF}}$ we use the following flag of complexity classes

$$\mathrm{NP}^{\mathrm{TQBF}} \subseteq \mathrm{NPSPACE} = \mathrm{PSPACE} \subseteq \mathrm{P}^{\mathrm{TQBF}}.$$

Justify each step.

(h) Explain why it is unlikely that a diagonalization argument could decide whether $\mathrm{P} \neq \mathrm{NP}$.

PROBLEM 4. (bonus) Is it true that $\mathrm{P}^{\mathrm{SAT}} = \mathrm{NP}^{\mathrm{SAT}}$?