Math 387 Group problems, Wednesday Week 10

PROBLEM 1. Review.

(a) Suppose that $f\colon \mathbb{N} \to \mathbb{N}$ where $f(n)$ is at least $O(\log(n))$. What does it mean to say *space constructible*?

(b) Name a function $f(n)$ that is in $O(n)$ but not $o(n)$. Show the relevant calculation.

(c) State the space hierarchy theorem.

(d) Where is space constructibility used in the proof of the space hierarchy theorem?

(e) For any two rational numbers $0 \leq a < b$, show that $n^a \in o(n^b)$ and hence, via the space hierarchy theorem, $\text{SPACE}(n^a) \subsetneq \text{SPACE}(n^b)$. Show the relevant calculation. (Here, we are using the fact that n^c is space constructible for any rational number c.)

(f) Savitch's theorem says $\text{NL} \subseteq \text{SPACE}(\log^2(n))$. Use this to prove that $\text{NL} \subsetneq \text{PSPACE}$. Show the relevant calculation.

(g) State the time hierarchy theorem.

(h) Prove that $\text{TIME}(n^2) \subsetneq \text{TIME}(n^3)$. Show the relevant calculation.

PROBLEM 2.

(a) We know that PATH is NL-complete, i.e., everything in NL is log space reducible to PATH. Prove that $\overline{\text{PATH}}$ is coNL-complete.

(b) We have seen that $\overline{\text{PATH}}$ is in NL. Explain why this implies NL = coNL.

PROBLEM 3.

(a) Prove that $\text{TIME}(2^n) = \text{TIME}(2^{n+1})$.

(b) Prove that $\text{TIME}(2^n) \subsetneq \text{TIME}(2^{2n})$.

PROBLEM 4.

(a) Explain why A_{NFA} is in NL.

(b) Prove that A_{NFA} is NL-complete by giving a reduction from PATH.