Math 387 Group problems, Monday Week 10

PROBLEM 1. Review.

(a) Why is PATH in NL?

(b) What is a log-space transducer?

(c) What does it mean to say a language A is log-space reducible to B?

(d) If $A \leq_L B$ and B is in L, why is A in L? (How does one get around the problem that if f is the mapping reduction, $f(w)$ my have length that is longer than $O(\log(n))$?

(e) Why is NL is in P?

PROBLEM 2. Let A be the language of properly-nested parentheses. For example, A contains (()) and ()(()()) but not ())(. Show that A is in L.

PROBLEM 3. A directed graph is *strongly connected* if for all ordered pairs of vertices (u, v) there is a directed path from u to v. Thus, there is a directed path between every pair of vertices in both directions. Let

$$SC = \{\langle G \rangle : G \text{ is a strongly connected directed graph}\}.$$

(a) Show that SC is in NL.

(b) Given a directed graph G and two vertices s and t of G. Create a new graph G' with the same vertices as G such that G' is strongly connected if and only if there is a path in G from s to t.

(c) Show that SC is NL-complete by reducing from PATH.