Problem 1.

- (a) Time complexity of a TM is defined as a function. What are the domain and codomain of that function? Precisely what does the function measure. (What is a *step*?)
- (b) Same question but for the time complexity of a NTM.
- (c) Same question but for the space complexity of a TM.
- (d) Same question but for the space complexity of a NTM.
- (e) How is space complexity affected if a tape cell is scanned more than once?
- (f) Are time and space complexity defined for all TMs and NTMs?

PROBLEM 2. Suppose that a TM runs in $O(n^2)$ space? Is there an upper bound on its time complexity? Explain.

PROBLEM 3. What is the relation between languages in P and languages in PSPACE? Explain.

PROBLEM 4. What is wrong with the following argument? If A and B are sets and $A \subseteq B$, then $B^c \subseteq A^c$. We know that NP \subseteq PSPACE. Therefore, coPSPACE \subseteq coNP. Since coPSPACE = PSPACE, we then have PSPACE \subseteq coNP.

PROBLEM 5. Consider the language

 $E_{\text{DFA}} = \{ \langle M \rangle : M \text{ is a DFA, and } L(M) = \emptyset \}.$

Is E_{DFA} in PSPACE? (Recall the state-marking algorithm in the text for deciding E_{DFA} .)

PROBLEM 6. Is PSPACE closed under the following four operations?

- (a) Union.
- (b) Concatenation.
- (c) Complementation.
- (d) Star.

For each operation under which PSPACE is closed, provide a proof.