PROBLEM 1. Sipser described a polynomial time reduction of 3SAT to CLIQUE.

(a) (i) Use the same idea to associate a graph to the boolean formula $\phi = (x \vee y) \wedge (\overline{x} \vee z) \wedge (y \vee \overline{z})$ so that the solutions in the formula correspond to 3-cliques in the graph.

(ii) Give some specific examples of solutions and their corresponding cliques.

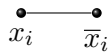
(b) Suppose you are given a 3SAT problem with k clauses. Construct the corresponding graph as described by Sipser. Is it possible for the graph to have a clique of size larger than k. Give a proof that is it not possible, or give an explicit example showing that it is possible.

PROBLEM 2. An undirected finite graph is 3-*colorable* if there is a way to assign one of three different colors to each of its vertices in such a way the no two adjacent vertices have the same color. Let

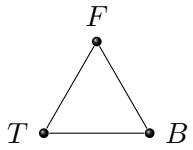
$$\text{3COLOR} = \{\langle G \rangle : G \text{ is a 3-colorable graph}\}.$$

In this problem, you are asked to give a polynomial time reduction of 3SAT to 3COLOR in order to show that 3COLOR is NP-complete.

To describe the reduction, let the colors be True, False, and Blank. Take one vertex for each literal x_i and one for its negation $\overline{x}_i$. We do not want both x_i and $\overline{x}_i$ to both be assigned True or both be assigned False. So add edges:

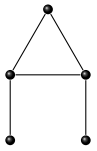


To make sure that no vertex corresponding to a literal is colored Blank, add the following triangle to the graph:



The vertices are labeled T, F, and B, as shown. (These are labels, not the colors True, False, Blank. However, given any 3-coloring, we can permute the colors to get another 3-coloring such that T, F, B are colored True, False, Blank, respectively). Now connect vertices representing literals to the vertex of the triangle labeled B.

The following structure serves as an "OR-gadget":



By adding two edges from the top vertex of the gadget: one to vertex B and one to vertex F, we can assure that in any 3-coloring, the top vertex will have the same color as vertex T.

(a) Use the pieces above to create a graph corresponding to the clause $x \vee \overline{y}$ in the variables x and y.

(b) Use the pieces above to create a graph corresponding to the clause $x \vee \overline{y} \vee z$ in the variables x, y and z.

(c) Describe the construction for a general formula in 3SAT.

PROBLEM 3. (bonus) Let $f_1, \ldots, f_m$ be a list of final exams, and let $s_1, \ldots, s_n$ be a list of students. Each student takes some specified subset of the final exams. Each final exam is a single time slot in length. The problem is to determine if these finals can be scheduled in only h non-overlapping time slots so that no student has two simultaneous exams. Show that this scheduling problem is NP-complete.