PROBLEM 1.

(a) What is a *computable function*? (Give a precise definition that includes stating the domain and codomain.).

(b) Let A, B be languages. What does it mean to say A is *reducible* to B? What does it mean to say A is *mapping reducible* to B?

(c) Why is A always reducible to $\overline{A}$?

(d) Give an example, with explanation, of a language that is reducible to its complement but not mapping reducible to its complement.

PROBLEM 2. Show that a language A is recognizable if and only if $A \leq_m A_{TM}$. (Divide your proof into two parts. In the first, assume that A is recognizable and provide an explicit mapping reduction. This will be a computable function $f\colon \Sigma^* \to \Sigma^*$. So f must account for every possible string, and its output must be a string.)

PROBLEM 3. Let $T = \{\langle M \rangle : M \text{ is a TM and } L(M) = L(M)^R\}$. Describe a reduction from A_{TM} in order to prove that T is undecidable. (Hint: mimic the proof of Theorem 5.3 in our text.)

PROBLEM 4. We have seen that A_{TM} is reducible to E_{TM}. Review the proof.

(a) Is E_{TM} mapping reducible to A_{TM}?

(b) (bonus) Is it reducible to A_{TM}?

PROBLEM 5. Is the language T from Problem 3 recognizable?

PROBLEM 6. (bonus) Let $S = \{\langle M \rangle : M \text{ is a TM and } L(M) = \{\langle M \rangle\}\}$. Show that neither S nor $\overline{S}$ is recognizable.