Math 387 Group problems, Wednesday Week 4

PROBLEM 1. Let

$$A_{\mathrm{TM}} = \{\langle M, w\rangle : M \text{ is a TM and } M \text{ accepts } w\}.$$

Review the following, trying not to consult outside sources.

(a) Is A_{TM} recognizable?

(b) Is A_{TM} decidable?

PROBLEM 2. Why is the class of Turing-recognizable languages not closed under complementation? Again, try to answer this as a group without consulting outside resources.

PROBLEM 3. Let

$$NH = \{\langle M, w\rangle : M \text{ will not halt on } w\}.$$

If NH decidable? Prove your answer.

PROBLEM 4. Let

$$E_{\mathrm{TM}} = \{\langle M\rangle : \ M \text{ is a TM and } L(M) = \emptyset\}.$$

Prove that E_{TM} is co-Turing-recognizable.

PROBLEM 5. Let $\Sigma = \{0, 1\}$. Determine, with explanation, which of the following are countable:

(a) The set of all languages, Σ^*.

(b) The set of all strings Σ^*.

(c) Is every regular language countable?

(d) Is the set of all regular languages countable?

PROBLEM 6. For any language A let

$$A^r = \{w^r : w \in A\}.$$

So A^r consists of the reversal of all words in A.

(a) If A is a decidable language, is A^r decidable?

(b) If A is a recognizable language, is A^r recognizable?

(c) Suppose that A is decidable. Is determining whether $A = A^r$ decidable? In other words, is there a Turing machine N that halts on all inputs, takes as input $\langle M\rangle$ where M is a Turing machine that halts on all inputs, and then accepts if $L(M) = L(M)^r$ and rejects, otherwise?

PROBLEM 7. (bonus) Write a program (in any programming language) that prints its own code to the screen. The program cannot interact with the file that contains the program. (It can't, for example, read or copy the file that contains its code.)