Math 387 Group problems, Monday Week 4

PROBLEM 1. Let $A = \{ \langle R, S \rangle : R \text{ and } S \text{ are regular expressions and } L(R) \subseteq L(S) \}$. Show that A is decidable.

PROBLEM 2. A useless state in a PDA is a state that is never entered on any input string. Let $L = \{ \langle P \rangle : P \text{ is a PDA with a useless state} \}$. Prove that L is decidable.

PROBLEM 3. Let A and B be DFAs. Let A have states Q_A and B have states Q_B . Show that if $L(A) \neq L(B)$, then there exists a word w of length less than or equal to $|Q_A||Q_B|$ that is accepted by one machine and not the other.

PROBLEM 4. Prove that if G is a CFG is in Chomsky normal form, then for any string w of length $|w| = n \ge 1$, exactly 2n - 1 steps are required for any derivation of w. (For the relevance of this problem, recall that our text uses it to prove that the acceptance question for CFG is decidable, i.e. the language $A_{\rm CFG} = \{\langle G, w \rangle : G \text{ is a CFG that generates } w\}$ is decidable.) Your proof will judged according to the clarity of its exposition—it should not require too much ingenuity on the part of the reader.

PROBLEM 5. (bonus) Let

 $E = \{ \langle M \rangle : M \text{ is a DFA that accepts some string with more 1s than 0s} \}.$ Is E decidable?